SEM-II
Hons (C IV: WAVES AND OPTICS)
L-2
Fraunhofer diffraction: Double slits

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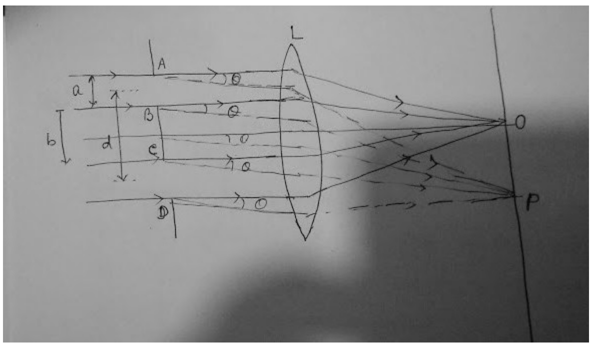
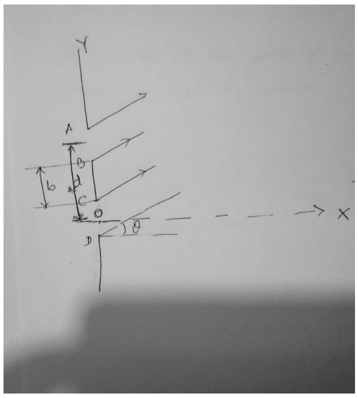
Let us now consider a parallel beam of monochromatic light of wavelength λ fall normally on a double slit (AB and CD). The distance between the slits is b and width of each slits a . Here each slit produces a diffracted beam and this difracted beams interfere with each other when brought to focus by lens.
So we shall get the diffraction pattern due to single slit on the screen and also the interference pattern on on the screen due to due to light waves coming from two slits.
Let O the origin of the coordinate system be considered at the center of the slit CD
Consider the element dy taken at a distance y from the origin.

The limit of integration for CD will be from $-\frac{a}{2}$ to $\frac{a}{2}$ and for AB will be from $d-\frac{a}{2}$ to $d+\frac{a}{2}$, where $d = a + b$





Now the displacement at any point p on the screen due the wavelets dy at any time t is given by

$$dz = KdySin2\pi(\frac{t}{T} - \frac{r}{\lambda} + \frac{ySin\theta}{\lambda}) \tag{0.1}$$

Hence the total displacement due to both slit can be obtained by integrating

$$z = K[\int_{-\frac{a}{2}}^{\frac{a}{2}} Sin2\pi(\frac{t}{T} - \frac{r}{\lambda} + \frac{ySin\theta}{\lambda})dy + \int_{d-\frac{a}{2}}^{d+\frac{a}{2}} Sin2\pi(\frac{t}{T} - \frac{r}{\lambda} + \frac{ySin\theta}{\lambda})dy] \tag{0.2}$$

$$z = Ka\frac{Sin\alpha}{\alpha}Sin2\pi(\frac{t}{T} - \frac{r}{\lambda}) - \frac{K\lambda}{2\pi Sin\theta})[Cos2\pi(\frac{t}{T} - \frac{r}{\lambda} + \frac{ySin\theta}{\lambda})]_{d-\frac{a}{2}}^{d+\frac{a}{2}} \tag{0.3}$$

$$z = Ka\frac{Sin\alpha}{\alpha}Sin2\pi(\frac{t}{T}-\frac{r}{\lambda}) - \frac{K\lambda}{2\pi Sin\theta})[Cos2\pi(\frac{t}{T}-\frac{r}{\lambda}+\frac{dSin\theta}{\lambda}+\frac{aSin\theta}{2\lambda}) - Cos2\pi(\frac{t}{T}-\frac{r}{\lambda}+\frac{dSin\theta}{\lambda}-\frac{aSin\theta}{2\lambda})] \tag{0.4}$$

$$z = Ka\frac{Sin\alpha}{\alpha}Sin2\pi(\frac{t}{T}-\frac{r}{\lambda}) + \frac{K\lambda}{\pi Sin\theta}[Sin2\pi(\frac{t}{T}-\frac{r}{\lambda}+\frac{dSin\theta}{\lambda})Sin\frac{\pi aSin\theta}{\lambda}] \tag{0.5}$$

$$z = Ka\frac{Sin\alpha}{\alpha}[Sin2\pi(\frac{t}{T}-\frac{r}{\lambda}+\frac{dSin\theta}{\lambda}) + Sin2\pi(\frac{t}{T}-\frac{r}{\lambda}] \tag{0.6}$$

$$z = 2Ka\frac{Sin\alpha}{\alpha}Sin2\pi(\frac{t}{T}-\frac{r}{\lambda}+\frac{dSin\theta}{2\lambda})Cos(\frac{\pi dSin\theta}{\lambda}) \tag{0.7}$$

$$z = 2Ka\frac{Sin\alpha}{\alpha}Cos\beta Sin2\pi(\frac{t}{T}-\frac{r}{\lambda}+\frac{dSin\theta}{2\lambda}) \tag{0.8}$$

$$z = A_\theta Sin2\pi(\frac{t}{T}-\frac{r}{\lambda}+\frac{dSin\theta}{2\lambda}) \tag{0.9}$$

Where $A_\theta = 2Ka\frac{Sin\alpha}{\alpha}Cos\beta$ and $\beta = \frac{\pi dSin\theta}{\lambda}$ and $\alpha = \frac{\pi aSin\theta}{\lambda}$
Hence the resultant disturbance is simple harmonic vibration with amplitude

$$A_\theta = 2Ka\frac{Sin\alpha}{\alpha}Cos\beta \tag{0.10}$$

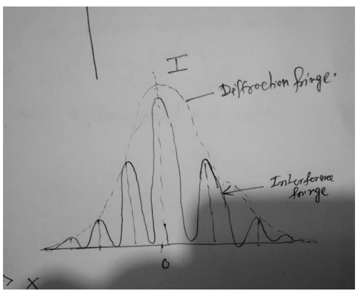
So the Intensity I in diffraction at p on the screen

$$I = A_\theta^2 = 4K^2a^2\frac{Sin^2\alpha}{\alpha^2}Cos^2\beta \tag{0.11}$$

$$I = 4I_0\frac{Sin^2\alpha}{\alpha^2}Cos^2\beta \tag{0.12}$$

Where $I_0 = K^2a^2$
This is the expression of resultant intensity due to a double slits diffraction .

Intensity Distribution Curve: Variation of I with θ due to double slit. In equation (1.50) the first factor $\frac{Sin^2\alpha}{\alpha^2}$ gives the diffraction pattern due to single slit whereas the factor $Cos^2\beta$ corresponds to the interference due to both slits.



1. <u>Condition for central maxima:</u>
When $\theta = 0$,$\alpha = 0$and $\beta = 0$
The intensity $I = 4I_0$ i. e. four times the intensity of the central maximum of the single slit diffraction pattern.

2.<u>Condition for minima:</u>

For the diffraction minima the intensity is zero for

$Sin\alpha = 0$ or $\alpha = \pm m\pi$, $[m = 1, 2, 3, 4, ..]$
i.e.

$$aSin\theta_m = \pm m\lambda \tag{0.13}$$

For interference minima $Cos^2\beta = 0$or $\beta = \pm(2n+1)\frac{\pi}{2}$,$(n = 0, 1, 2, 3, ...)$ So

$$(a+b)Sin\theta_n = \pm(n+\frac{1}{2})\lambda \tag{0.14}$$

This is the condition for interference minima.

2.<u>Condition for maxima:</u>

The interference maxima occurs when $Cos^2\beta = 1$ or $\beta = \pm n\pi$,$(n = 0, 1, 2, 3, ..)$
So

$$(a+b)Sin\theta_n = \pm n\pi \tag{0.15}$$

This is the condition for interference maxima.
So in double slit pattern there will be a number of alternate dark and bright interference fringes within each diffraction pattern.

Missing order of interference maxuma:
If the conditions of diffraction minima and the conditions for maxima of interference pattern are satisfied simultaneously for the same value of angle of diffraction θ, corresponding interference maxima will not be visible due to non availability of light at that precise position. Which is called missing order.

$$\frac{a+b}{a} = \frac{n}{m} \tag{0.16}$$

Where condition for diffraction minima $aSin\theta = m\lambda$in the single slit diffraction pattern and condition for maxima of interference pattern $(a+b)Sin\theta = n\lambda$.

Problem-1 Find the missing order for a double slit Fraunhofer pattern if the width of each slit is 0.15mm and they are separated by a distance of 0.60 mm.

Missing orders are obtained when interference maxima and diffraction mainima corresponds to the same value of diffraction angle θ.
So
$(a+b)Sin\theta = n\lambda$

$aSin\theta = m\lambda$
Hence

$$\frac{a+b}{a} = \frac{n}{m} \tag{0.17}$$

Hence

$$\frac{n}{m} = \frac{0.15+0.60}{0.15} = 5 \tag{0.18}$$

$$n = 5m \tag{0.19}$$

For the values of $m = 1, 2, 3, \quad n = 5, 10, 15$ etc.
SO, the $5th$,$10th$,$15th$ etc. orders of interference maxima will be missing in the diffraction pattern.